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R^2 corrections for 5D black holes and rings

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Abstract: We study higher-order corrections to two BPS solutions of 5D supergravity, namely the supersymmetric black ring and the spinning black hole. Due to our current relatively limited understanding of F-type terms in 5D supergravity, the nature of these corrections is less clear than that of their 4D cousins. Effects of certain R^2 terms found in Calabi-Yau compactification of M-theory are specifically considered. For the case of the black ring, for which the microscopic origin of the entropy is generally known, the corresponding higher order macroscopic correction to the entropy is found to match a microscopic correction, while for the spinning black hole the corrections are partially matched to those of a 4D $D0-D2-D6$ black hole.

Keywords: M-Theory, Black Holes in String Theory, D-branes, Topological Strings.

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1. Introduction

Recently a surprisingly powerful and precise relationship has emerged between higher dimension F-terms in the 4D effective action for $\mathcal{N} = 2$ string theory (as captured by the topological string [1]) and the (indexed) BPS black hole degeneracies [2, 3]. Even more recently [4] a precise relationship has been conjectured between the 4D and 5D BPS black hole degeneracies. This suggests that there should be a direct relationship between higher dimension terms in the 5D effective action and 5D degeneracies which does not employ four dimensions as an intermediate step. Five dimensions is in many ways simpler than four so such a relation would be of great interest. It is the purpose of this paper to investigate this issue.

The 4D story benefitted from a well understood superspace formulation [5, 6]. The relevant supersymmetry-protected terms are integrals of chiral superfields over half of superspace and can be classified. In 5D the situation is quite different (see e.g. [7]). There is no superfield formulation and we do not have a general understanding of the possible supersymmetry-protected terms. In general, the uplift to 5D of most of the 4D F-terms vanishes. However, the area law cannot be the exact answer for the black hole entropy (for one thing it doesn't give integer numbers of microstates!) so there must be some kind of perturbative supergravity corrections.

As a first step towards a more general understanding, in this paper we will study the leading order entropy correction arising from R^2 terms, which are proportional to the 4D Euler density. Such terms give the one loop corrections in 4D, and - unlike the higher order terms - do not vanish upon uplift to 5D. They are also of special interest as descendants of the interesting 11D R^4 terms [8, 9]. These terms correct the entropy of both the 5D black ring [10] and the 5D BMPV spinning black hole [11]. We find that the macroscopic black

ring correction matches, including the numerical coefficient, a correction expected from the microscopic analysis of [12]. For the BMPV black hole, we find the correction matches, to leading order, one expected from the 4D-5D relation conjectured in [4].

The next section derives the R^2 corrections to the 5D entropy as horizon integrals of curvature components using Wald's formula. Section 3 evaluates this formula for the black ring, while section 4 evaluates it for BMPV. Section 5 contains a brief summary.

2. Wald's formula in 5D

In this section we will use Wald's formula to derive an expression for R^2 corrections to the 5D entropy.

The Einstein-frame low energy effective action for the compactification of M-theory on a Calabi-Yau threefold CY_3 down to five dimensions contains the terms [13]

$$I_0+\Delta I=-\frac{1}{32\pi^2}\int d^5x\sqrt{|g_5|}R^{(5)}-\frac{1}{2^9\cdot 3\pi^2}\int d^5x\sqrt{|g_5|}c_{2A}Y^A(R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}-4R_{\alpha\beta}R^{\alpha\beta}+R^2) \tag{2.1}$$

in units in which $G_5=2\pi$ (for compactification on a circle of unit radius, this choice leads to $G_4=1$ and hence facilitates 4D/5D comparisons). Here Y^A, $A=1,\dots n_V$ are scalar components of vector multiplets. They are proportional to the Kähler moduli of CY_3, normalized so that

$$D_{ABC}Y^AY^BY^C=1. \tag{2.2}$$

c_{2A} are the components of the second Chern class of CY_3 and D_{ABC} the corresponding intersection numbers. The R^2 term in ΔI arises from dimensional reduction of the much studied R^4 term [8, 9] in eleven dimensions. It is also the uplift from four dimensions of an F term whose coefficient is computed by the $N=2$ topological string on CY_3 at one loop order [1].

When we add R^2 corrections to the action the entropy is no longer given by the area law; instead, we need to use the more general formula found by Wald [14]

$$S_{\rm BH}=2\pi\int_{\rm Hor}\frac{\partial\mathcal{L}}{\partial R_{\mu\nu\rho\sigma}}\epsilon_{\mu\nu}\epsilon_{\rho\sigma} \tag{2.3}$$

where, $\epsilon_{\alpha\beta}$ is the binormal to the horizon, defined as the exterior product of two null vectors normal to the horizon and normalized so that $\epsilon_{\alpha\beta}\epsilon^{\alpha\beta}=-2$. We can then identify two types of first-order corrections implied by this formula:

- modifications to the area law due to the additional terms in the action - these terms are evaluated using the zeroth order solutions for the metric and the other fields.
- modification of the area due to the change of the metric on the horizon, which follows from the fact that adding extra terms to the action may change the equations of motion.

In 4D, the second type of modification is absent at leading order for this particular R^2 form of ΔI obtained by reduction of (2.1) [15]. This and the 4D-5D agreement we find

to leading order suggest that this may be the case in 5D as well. In order to understand all R^2 corrections to the entropy this should be ascertained by direct calculation. In the following we consider only the first type of modification.

The corresponding correction to the entropy is then (see also [16])

$$\Delta S = -\frac{4\pi c_2 \cdot Y}{2^9 \cdot 3\pi^2} \int_{\rm Hor} d^3x \sqrt{h} \, (\, R_{\mu\nu\rho\sigma}\epsilon^{\mu\nu}\epsilon^{\rho\sigma} - 4 \, R_{\mu\rho}g_{\nu\sigma}\epsilon^{\mu\nu}\epsilon^{\rho\sigma} + \, R\epsilon_{\mu\nu}\epsilon^{\mu\nu}) \tag{2.4}$$

where h is the induced metric on the horizon and the moduli are fixed at their attractor values. In the following we will evaluate this correction for the spinning black hole and black ring solutions.

3. The black ring

The black ring solution was discovered in [10] and its entropy understood from a microscopic perspective in [12]. It represents a supersymmetric solution to 5D supergravity coupled to a number of abelian vector (and hyper)multiplets that describes a charged, rotating black ring. It is characterized by electric charges q_A, magnetic dipole charges p^A, and the angular momentum around the ring, J_ψ. The macroscopic entropy formula for the black ring can be written in the suggestive form

$$S_{\rm BR} = 2\pi \sqrt{\frac{c_L \hat{q}_0}{6}} \tag{3.1}$$

where, in terms of the macroscopic charges,

$$c_L = 6D = 6D_{ABC}p^A p^B p^C \tag{3.2}$$

D_{ABC} being (one sixth) the intersection numbers of the Calabi-Yau, and

$$\hat{q}_0 = -J_\psi + \frac{1}{12} D^{AB} q_A q_B + \frac{c_L}{24} \tag{3.3}$$

where D^{AB} is the inverse of $D_{AB} \equiv D_{ABC}p^C$. The microscopic origin of the entropy is from the quantum degeneracy of a 2D CFT with central charge c_L and left-moving momentum $\hat{q}_0$ available for distribution among the oscillators. The last term in (3.3) is ascribed to the left moving zero point energy.

3.1 Macroscopic entropy correction

Now we evaluate the correction to the black ring entropy induced by ΔI. Due to the 5D attractor mechanism [17] the moduli take the horizon values

$$Y^A = \frac{p^A}{D^{\frac{1}{3}}} \tag{3.4}$$

Next, all we need to do is to find the binormal to the horizon for the black ring metric, evaluate the relevant curvature terms at the horizon, and integrate. We obtain[1]

$$\Delta S_{\rm BR} = \frac{\pi}{6}\, c_2 \cdot p \, \sqrt{\frac{\hat{q}_0}{D}} \tag{3.5}$$

3.2 Microscopic entropy correction

The microscopic entropy comes from M5 branes wrapping 4 cycles associated to p^A in CY_3. As shown in [15], these are described by a CFT with left-moving central charge

$$c_L = 6D + c_2 \cdot p \tag{3.6}$$

In [12] the leading entropy at large charges was microscopically computed using the leading approximation (3.2) to c_L at large charges. Subleading modifications should arise from using the exact formula (3.6) in (3.1). This leads to

$$\Delta S_{\rm BR} = \frac{\pi}{6}\, c_2 \cdot p \, \sqrt{\frac{\hat{q}_0}{D}} + \frac{\pi}{24}\, c_2 \cdot p \, \sqrt{\frac{D}{\hat{q}_0}} + \dots \tag{3.7}$$

The first term comes from correcting c_L in (3.1), while the second comes from correcting the zero point shift in (3.3). We see that the macroscopic R^2 correction matches precisely the first term. We do not understand the matching of the second term, but note that it is subleading in the regime $\hat{q}_0 \gg D$, where Cardy's formula is valid.

4. The BMPV black hole

Let us now turn to BMPV - the charged rotating black hole in 5D characterized by electric charges q_A and angular momenta J in $\mathrm{SU}(2)_L$. Its leading macroscopic entropy is given by

$$S_{\rm BMPV} = 2\pi\sqrt{Q^3 - J^2} \tag{4.1}$$

where

$$Q^{\frac{3}{2}} = D_{ABC} y^A y^B y^C \tag{4.2}$$

where the y^A's are determined from

$$q_A = 3 D_{ABC} y^B y^C \tag{4.3}$$

[1]Note that in the following we have employed the following relationships between various quantities used in this paper, in [10] and in [11]:

$$Q^{\rm eemr} = (16\pi G)^{\frac{2}{3}} \mu^{\rm bmpv} = \left(\frac{4G}{\pi}\right)^{\frac{2}{3}} q$$

$$q^{\rm eemr} = \left(\frac{4G}{\pi}\right)^{\frac{1}{3}} p$$

$$J = J_{\rm eemr} = 16\pi J_{\rm bmpv} = 4\pi^2 \mu\omega$$

The value of Newton's constant used in [11] is $G_5 = (16\pi)^{-1}$, so we needed to rescale their metric by $(16\pi G)^{\frac{2}{3}}$ in order to get ours. Also recall we are setting $G = 2\pi$ in the text.

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We find the correction to this entropy following from the application of Wald's formula to (2.1) to be

$$\Delta S_{\rm BMPV} = -\frac{\pi}{24}\sqrt{Q^3 - J^2}\; c_2 \cdot Y \left(-\frac{3}{Q} - \frac{J^2}{Q^4}\right) = \frac{\pi}{6}\mathcal{A}\; c_2 \cdot Y \left(\frac{1}{Q} - \frac{\mathcal{A}^2}{4Q^4}\right) \tag{4.4}$$

where we defined $\mathcal{A} = \sqrt{Q^3 - J^2}$ and the moduli fields take the horizon values[2]

$$Y^A = \frac{y^A}{Q^{\frac{1}{2}}} \tag{4.5}$$

In general, the microscopic origin of the entropy for the 5D spinning black holes in M-theory on CY_3 (unlike for black rings) is not known,[3] so we will not try herein to understand the microscopic origin of $\Delta S_{\rm BMPV}$. We will however compare it to corresponding corrections in 4D and the topological string partition function. As argued in [4], the exact 5D BMPV entropy is equal to the entropy of the $D6-D2-D0$ system in 4D, with the same 2-brane charges q_A, $D6$-brane charge $p^0 = 1$, and $D0$-brane charge $q_0 = 2J$. In the same paper, the following relationship for the partition functions of 5D black holes, 4D black holes and consequently of the topological string - see [3] -was conjectured

$$Z_{5D}(\phi^A, \mu) = Z_{4D}(\phi^A, \phi^0 = \frac{\mu}{2} + i\pi) = \left| Z_{\rm top}\left(g_{\rm top} = \frac{8\pi^2}{\mu}, t^A = \frac{2\phi^A}{\mu}\right)\right|^2 \tag{4.6}$$

where ϕ^A are the electric potentials conjugate to q_A, while ϕ^0 is conjugate to q_0 in 4D, and $Re\mu = (\mu + 2\pi i)$ to J in 5D. The absolute value in the last expression is defined by keeping ϕ^0 real. With this in mind, we can start from $F_{\rm top}$ - the topological string amplitude- and compute the entropy of the BMPV (including first order corrections) as follows. Up to one-loop order $F_{\rm top}$ is

$$\begin{aligned} F_{\rm top} &= \frac{i(2\pi)^3}{g_{\rm top}^2} D_{ABC} t^A t^B t^C - \frac{i\pi}{12} c_{2A} t^A \\ &= \frac{i}{\pi}\frac{D_{ABC}\phi^A\phi^B\phi^C}{\mu} - \frac{i\pi}{6}\frac{c_{2A}\phi^A}{\mu} \end{aligned} \tag{4.7}$$

The entropy of the black hole is given by the Legendre transform of

$$\mathcal{F}(\phi^A, Re\mu) = \ln Z_{\rm BH} = F_{\rm top} + \bar{F}_{\rm top} \tag{4.8}$$

To first order we have

$$\mathcal{F} = -\frac{1}{\pi^2}\frac{D_{ABC}\phi^A\phi^B\phi^C - \frac{\pi^2}{6}c_{2A}\phi^A}{(\frac{Re\mu}{2\pi})^2 + 1} \tag{4.9}$$

[2]From now on we will take 'Y^A' to mean the horizon value of the modulus field Y^A.

[3]It is of course known for $\mathcal{N} = 4$ compactifications [18, 11], so it would be interesting to interpret the macroscopic correction for that case.

which gives

$$q_A = \frac{1}{\pi^2}\frac{3D_{ABC}\phi^B\phi^C - \frac{\pi^2}{6}c_{2A}}{(\frac{Re\mu}{2\pi})^2+1} \tag{4.10}$$

$$J = -\frac{Re\mu}{2\pi^4}\,\frac{D_{ABC}\phi^A\phi^B\phi^C - \frac{\pi^2}{6}c_{2A}\phi^A}{\left((\frac{Re\mu}{2\pi})^2+1\right)^2}$$

and therefore

$$S = 2\pi\sqrt{Q^3 - J^2}(1 + \frac{1}{12}\frac{c_{2A}Y^A}{Q} + \ldots) \tag{4.11}$$

where the ... stand for higher order corrections in $|g_{\rm top}|^2 = 16\pi^2\mathcal{A}^2/Q^3$.

We see that the 5D R^2 corrections (4.4) to the entropy do not exactly match the 4D corrections (4.11). This is possible of course because dimensional reduction of the 5D R^2 gives the 4D R^2 term plus more terms involving 4D field strengths. However we also see that the mismatch is subleading in the expansion in $g_{\rm top}$, and we can therefore conclude that the 5D R^2 term captures the subleading correction to the area law.

5. Summary

We have shown that higher dimension corrections to the 5D effective action do give corrections to the black hole/black ring entropy just as in 4D, but that the 5D situation is currently under much less control than the 4D one. Some leading order computations were performed and found to give a partial match between macroscopic and microscopic results. We hope these computations will provide useful data for finishing the 5D macro/micro story.

Note added in proof: We would like to draw attention to [19], which may explain the matching of the subleading term in equation (3.7).

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